A statistical framework for extracting time-varying oscillations from neural data

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Summary

To study the neural basis of brain states, such as states of consciousness during the induction of anesthesia, and how they relate to interactions between brain regions, it is important to track the dynamics of spectral power and time-domain quantities, such as phases, in specific frequency bands of the neural data. Bandpass filtering offers a simple and popular approach to this problem. However, the absence of a corresponding statistical generative model makes it difficult to quantify the uncertainty of power or phase estimates and, as a consequence, to assess the statistical significance of the subsequent inferences. Approaches to mitigate these limitations partition time series into independent, approximately stationary intervals, and fit a stationary generative model to each interval. The assumption of independence of successive intervals results in distortion/discontinuity around the boundaries, which can lead to erroneous scientific interpretation and phenomena such as the "phase slip". We propose a statistical generative model, termed the piecewise locally stationary oscillatory (PLSO) model, that decomposes a time series with slowly-varying spectra into piecewise-stationary oscillatory components. PLSO models the signal dynamics for the entire data, preventing artifacts from data partitioning, and providing estimates and uncertainty for time-domain quantities of interest. In addition, PLSO can capture spectral dynamics in the time-frequency domain. We first apply PLSO to rat hippocampal LFP data from an open field task. By leveraging PLSO's ability to conduct inference on the entire time series, we show that the coupling of population spikes to theta phase observed in previous studies is indeed scientifically significant. Moreover, PLSO successfully identifies the prominent spectral components, specifically the theta band, and denoises the signal without introducing boundary artifacts. We also apply PLSO to EEG from humans under propofol anesthesia and show that PLSO can capture smooth spectral dynamics while removing noise/motion artifacts.

Additional Details

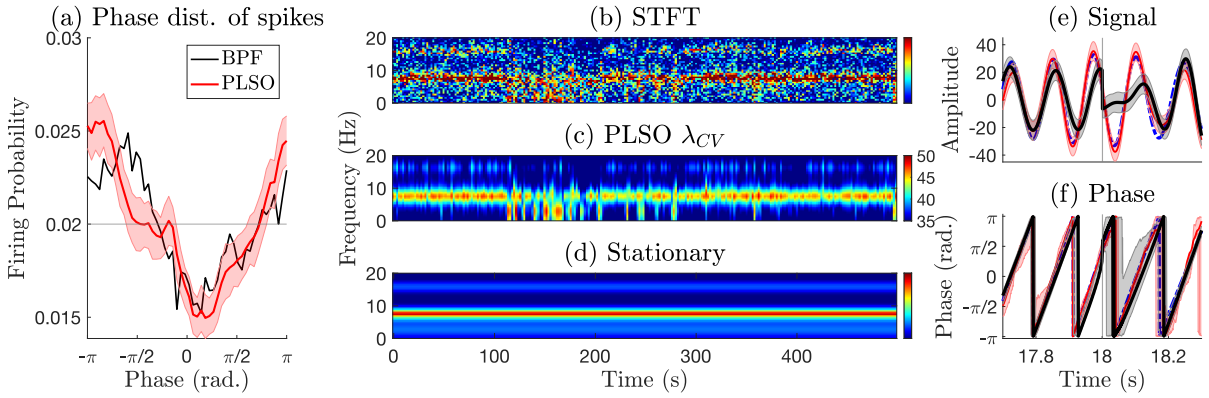

Figure 1: Hippocampal data analyses. (a) Theta phase distribution of population spikes, based on BPF (black), PLSO estimate with 95 % CI from 200 posterior samples (red). Horizontal gray indicates uniform distribution. (b-d) LFP spectrogram (in dB) (b) STFT (c) PLSO with λ_{CV} (d) Stationary. (e) Time-domain signal (f) phase and interval boundary (vertical gray), estimated with BPF (dotted blue), STFT-reg. (black), and PLSO (red).

Statistical inference framework for the theta phase distribution of spikes We apply PLSO to rat hippocampal LFP data, sampled at rate $f_s = 1{,}250$ Hz [1], assuming $J = 6$ oscillatory processes and 2-second stationary intervals. The theta oscillation is believed to play a role in coordinating the firing of neurons in the entorhinal-hippocampal system. Fig. 1(a) shows the theta phase distribution of hippocampal population neuron spikes. The PLSO-estimated distribution (red) confirms reports based on bandpass filtering (BPF, black) that the spikes show a strong preference for a specific theta ($5 \sim 10$ Hz) phase. For this result to be statistically conclusive, we need to determine whether this preference is *statistically significant* and quantify the uncertainty of the estimate of the phase distribution. Although hypothesis tests, such as the Rayleigh test, can be used to assess statistical significance, they do not

indicate *which* phase is preferred. With BPF, computing the uncertainty around phase estimates remains a challenge. PLSO solves both of these problems, using Monte Carlo (MC) trajectory samples from the posterior distribution of phase given the data. This lets us obtain both point estimates of phase, as well as the credible interval around these estimates. Since the 95% CI does not include the uniform distribution (horizontal gray), we can conclude that the phase preference is indeed statistically significant, with a preference for π.

Estimation of spectral/time-domain dynamics for Hippocampal LFP Fig. 1(c-d) shows the PLSO-estimated spectrogram. PLSO with cross-validated hyperparameter λ, λ_{CV}, identifies sustained power at 7.6 Hz and weaker bursts at 3.0/15.9 Hz. As a reference, Fig. 1(d) shows the results for assuming stationarity for the entire data, obliviating the need for partitioning. The time-domain estimates do not suffer boundary artifacts (not shown), albeit at the cost of failing to capture the spectral dynamics present. Fig. 1(e-f) shows a segment of the estimated signal and phase near a boundary for the theta band component, in the time domain. As baselines, we use regularized STFT (STFT-reg.), which imposes stochastic continuity on the STFT coefficients, and BPF, which provides a proxy for the ground truth. While the STFT-reg. (black) and PLSO (red) estimates closely follow the BPF (dotted blue) estimate, STFT-reg. exhibits discontinuity/distortions near the boundary, resulting in "phase slip". The average phase discontinuity at the boundary for STFT-reg. is 26.8 degrees, compared to 2.2/2.4 degrees for BPF/PLSO. Since the theta band roughly progresses $2.16\,(=\ 7.5(\text{Hz})\ \times\ 360/1250\,(\text{Hz}))$ degrees per sample, we conclude that window-based methods introduce significant boundary artifacts.

PLSO generative model We partition the data $\mathbf{y} \in \mathbb{R}^K$, into M non-overlapping stationary intervals of length N, i.e., $K = MN$. PLSO models the data as a superposition of J latent oscillatory processes, $\mathbf{z}_j \in \mathbb{C}^K$, with $\mathbf{z}_{j,mN+n}$ denoting n^{th} sample in the m^{th} interval. Each $\mathbf{z}_j \in \mathbb{C}^K$ models a quasi-periodic signal centered at frequency $\omega = \omega_j$, with the peak power and bandwidth determined by $\sigma^2_{j,m}$ and l_j, respectively, as shown in Fig. 2(a). PLSO is given as a two-level state-space model

$$\mathbf{z}_{j,mN+n} = \exp\left(-(f_s \cdot l_j)^{-1}\right) \exp\left(-i\omega_j\right) \mathbf{z}_{j,mN+(n-1)} + \sigma^2_{j,m}(1 - \exp(-2(f_s \cdot l_j)^{-1}))\varepsilon_{j,mN+n}, \tag{1a}$$

$$\log(\sigma^2_{j,m}) = \log(\sigma^2_{j,m-1}) + (1/\lambda)\,\eta_{j,m}, \quad \text{where } \eta_{j,m}, \nu_{mN+n} \sim \mathcal{N}(0,1) \text{ and } \varepsilon_{j,mN+n} \sim \mathcal{CN}(0,1) \tag{1b}$$

with $\mathbf{y}_{mN+n} = \sum_{j=1}^{J} \Re\left(\mathbf{z}_{j,mN+n}\right) + \nu_{mN+n}$. The *sample-level* model (Eq.1a) captures the sequential dynamics of $\mathbf{z}_{j,k}$ *within* and *across* different intervals. The *window-level* model (Eq.1b) imposes smoothness on $\sigma^2_{j,m}$. PLSO with λ_{CV} captures smooth spectral dynamics without overfitting to data.

Estimation/Inference We perform estimation/inference on the posterior $p(\{\mathbf{z}_j\}_j, \{\sigma^2_{j,m}\}_{j,m} \mid \mathbf{y}, \theta) = p(\{\sigma^2_{j,m}\}_{j,m} \mid \mathbf{y}, \theta) \cdot p(\{\mathbf{z}_j\}_j \mid \{\sigma^2_{j,m}\}_{j,m}, \mathbf{y}, \theta)$, with $\theta = \{\lambda, \sigma^2_\nu, \{l_j\}_j, \{\omega_j\}_j\}$. Using the proximal method for the nonconvex objective, we first maximize $\log p(\{\sigma^2_{j,m}\}_{j,m} \mid \mathbf{y}, \theta)$ with respect to θ and $\{\sigma^2_{j,m}\}_{j,m}$. Then, we perform inference on $p(\{\mathbf{z}_j\}_j \mid \{\sigma^2_{j,m}\}_{j,m}, \mathbf{y}, \theta)$, which is a Gaussian distribution. We can use the Kalman filter/smoother to estimate the mean trajectory and CI. Time-domain quantities, such as phase, can be estimated using the MC samples from the forward-filtering backward-sampling algorithm.

Estimation of time-varying spectra in anesthesia EEG We apply PLSO to propofol anesthesia EEG ($f_s = 250$ Hz), with $J = 9$ and 4-second stationary intervals, as in Fig. 2(b-c). PLSO identifies strong slow ($0.1 \sim 2$ Hz) and alpha oscillations ($8 \sim 15$ Hz), both well-known signatures of propofol-induced unconsciousness. We also observe that PLSO with λ_{CV} enables the estimation of smooth spectral dynamics, as well as the removal of spurious movement-artifacts (vertical lines) and noise.

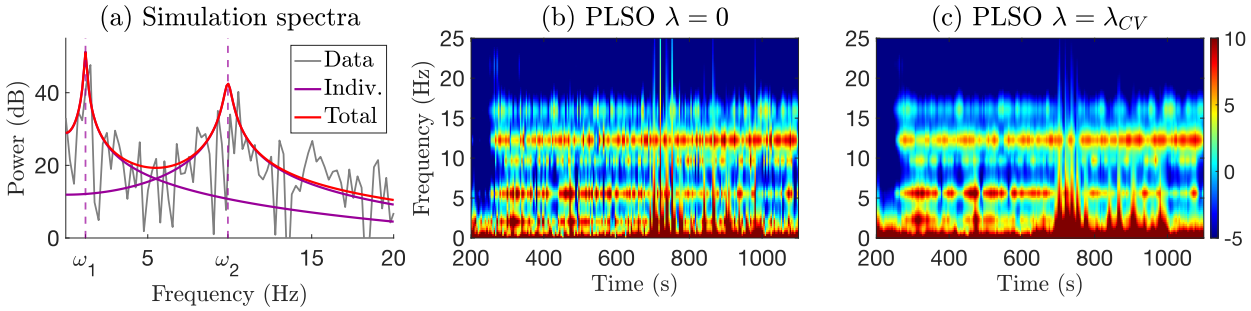

Figure 2: (a) Simulation spectra. Spectrum of the data (gray), PLSO-estimated components for $J = 2$ (purple) and their sum (red). (b) Spectrogram of EEG (in dB) under propofol anesthesia. (a) PLSO with $\lambda = 0$ (b) PLSO with λ_{CV}.

[1] K. Mizuseki et al., Theta oscillations provide temporal windows for local circuit computation in the entorhinal-hippocampal loop. *Neuron*, vol. 64, pp. 267-280, 2009.